

# Length of Stay Outlier Detection through Cluster Analysis: A Case Study in Pediatrics

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## Abstract

The increasing availability of detailed inpatient data is enabling the development of data-driven approaches to provide novel insights for the management of Length of Stay (LOS), an important quality metric in hospitals. This study examines clustering of inpatients using clinical and demographic attributes to identify LOS outliers and investigates the opportunity to reduce their LOS by comparing their order sequences with similar non-outliers in the same cluster. Learning from retrospective data on 353 pediatric inpatients admitted for appendectomy, we develop a two-stage procedure that first identifies a typical cluster with LOS outliers. Our second stage analysis compares orders pairwise to determine candidates for switching to make LOS outliers similar to non-outliers. Results indicate that switching orders in homogeneous inpatient sub-populations within the limits of clinical guidelines may be a promising decision support strategy for LOS management.

*Keywords:* Machine Learning; Clustering; Health Care; Length of Stay

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## 1. Introduction

Length of Stay (LOS) is an important quality metric in hospitals that has been studied for decades Kim and Soeken (2005); Tu et al. (1995). However, the increasing digitization of healthcare with Electronic Health Records and other clinical information systems is enabling the collection and analysis of vast amounts of data using advanced data-driven methods that may be particularly valuable for LOS management Gartner (2015); Saria et al. (2010). When patients are treated in hospitals, information about each individual is necessary to perform optimal treatment and scheduling decisions, with the detailed data being documented in the current generation of information systems. Recent research has highlighted that resource allocation decisions can be improved by scheduling patient admissions, treatments and discharges at the right time Gartner and Kolisch (2014); Hosseinifard et al. (2014) while machine learning methods can improve resource allocation decisions and the accuracy of hospital-wide predictive analytics tasks Gartner et al. (2015a). Thus, using data-driven analytic methods to understand length of stay (LOS) variations and exploring opportunities for reducing LOS with a specific focus on LOS outliers is the goal of this study.

Using retrospective data on 353 inpatients treated for appendectomy at a major pediatric hospital, we first carry out a descriptive data analysis and test which (theoretical) probability distribution best fits our length of stay data. The results reveal that our data matches observations from the literature. In a first stage cluster analysis, we identify one potential outlier cluster while a descriptive analysis using box plot comparisons of this cluster vs. the union of patients assigned to all other clusters supports this hypothesis. In a second clustering stage, we analyse the patient sub-population who belongs to that outlier cluster and provide order prescription behaviour insights. More specifically, on a pairwise comparison, we describe which orders are likely to be selected in the outlier population vs. ones that are deselected in the non-outlier population and vice versa. Our findings reveal that four order items are not prescribed in the outlier population while in the non-outlier sub-population, these orders were prescribed. On the other hand, 51 orders were prescribed for the outlier patients which are not enabled in the non-outlier population. These novel data-driven insights can be offered as suggestions for clinicians to apply new evidence-based, clinical guideline-compliant opportunities for LOS reduction through healthcare analytics.

## 2. Related Work

Clustering algorithms and other machine learning approaches are discussed in Baesens et al. (2009); Jain (2010); Meisel and Mattfeld (2010); Olafsson et al. (2008) including an overview of operations research (OR) techniques applied to data mining. Mathematical programming and heuristics for clustering clinical activities in Healthcare Information Systems has been applied in Gartner et al. (2015b) while the identification of similar LOS groups has been studied by El-Darzi et al. (2009). Similar to our problem, the authors study the application of approaches to cluster patient records with similar demographic and clinical conditions. Using a stroke dataset, they compare the performance of Gaussian Mixture Models,  $k$ -means clustering and a two-step clustering algorithm. Determining cluster centers for patients in the Emergency Department (ED) is studied by Ashour and Okudan Kremer (2014). Having defined similar patient clusters, they study the improvement on patient routing decisions based on the clusters. Similarly, Xu et al. (2014) focuses their clustering problem on the ED. Their objective is to cluster patients to resource consumption classes determined by length of treatment while patient demographics are taken into consideration.

The approaches proposed in our paper can be categorized and differentiated from the literature of clustering in length of stay management as follows: Using a descriptive data analysis we provide an overview about the characteristics of our length of stay data. Fitting several distribution types and parameters of the theoretical probability distribution, we underline the skewed property of the probability distribution from our data. In a next step, we define homogeneous patient groups with respect to demographic, clinical attributes and length of stay outliers. Having learned homogeneous groups of inpatients, we evaluate patient orders within the group that potentially contains length of stay outliers and may be responsible for increasing LOS in that group. In conclusion, this study may be considered to be the first to link the discovery of similar clinical and demographic attributes in appendectomy inpatients while, within length of stay outlier clusters, we evaluate possibilities for switching of orders and how they potentially reduce the number of LOS outliers.

### 3. Methods

Let  $\mathcal{P}$  denote a set of individuals (hospital inpatients) and let  $\mathcal{K}$  denote the set of clusters to which these individuals can be grouped. For each inpatient  $p \in \mathcal{P}$ , we observe a set of attributes  $\mathcal{A}$  during the patient’s LOS. Let  $\mathcal{V}_a$  denote the set of possible values for attribute  $a \in \mathcal{A}$  and let  $v_{p,a} \in \mathcal{V}_a$  denote the value of attribute  $a$  for inpatient  $p$ . In the following, we will describe how we label patients as LOS outliers, followed by a two-stage clustering approach: The first stage assigns patients’ attributes to homogeneous clusters while clusters with high likelihood to contain LOS outliers can be identified. In a second stage, we filter patients assigned to these clusters and evaluate which patient orders may be switched to reduce length of stay in the LOS outlier patient sub-population. The section closes with an illustrative example.

Given the observed LOS of patient  $p \in \mathcal{P}$ , denoted by  $l_p$ , the 25 and 75 percentile of the LOS distribution denoted by  $q^{25}$  and  $q^{75}$ , respectively then we assign a patient the flag “outlier” using the following expression (Pirson et al. (2006)):

$$o_p = \begin{cases} 1, & \text{if } l_p > q^{75} + (1.5 \cdot (q^{75} - q^{25})) \\ 0, & \text{otherwise} \end{cases}. \quad (1)$$

Now, let  $\mathcal{A}^{\text{dem}}$  denote the set of binary demographic attributes and let  $v_{a,p} \in \{0, 1\}$  denote the attribute value of demographic attribute  $a \in \mathcal{A}^{\text{dem}}$  of patient  $p \in \mathcal{P}$ . Let  $\mathcal{K} := \{1, 2, \dots, K\}$  be a set of integers with maximum  $K$  which will be used for cluster indexing. Our objective is to find cluster centers of patient attributes in order to minimize deviations of each patient’s attribute values with the ones of the cluster centres. One algorithm that minimizes this objective is the k-means clustering algorithm Jain (2010). The algorithm is a method of vector quantization. It seeks to partition observations into clusters in which each observation belongs to the cluster with the nearest mean which serves as a prototype of the cluster.

Once we have found patients with similar clinical, demographic and LOS characteristics, we wish to separate patients within the cluster that has the highest likelihood to contain LOS outliers. In this stage, we extract patients with these attributes and evaluate the order prescription behaviour for these patients between outliers and the false positively clustered outliers which actually belong to the group of non-outliers. Orders prescribed by clinicians to

patients are, for example, the application of drugs, examinations and therapies. Having determined patients with high likelihood of belonging to the group of outliers, we introduce a set  $\mathcal{A}_k^{\text{off} \rightarrow \text{on}}$  for cluster  $k \in \mathcal{K}$  which allows experts to evaluate orders which were switched off for outlier patients and were switched on for non-outliers. The set is determined by  $\mathcal{A}_k^{\text{off} \rightarrow \text{on}} := \{a \in \mathcal{A}^{\text{order}} | v_{a,p^*(p)} - v_{a,p} = 1 \quad \forall k \in \mathcal{K}, p \in \mathcal{P}_k^{\text{out}}\}$ . Similarly, we introduce the set  $\mathcal{A}_k^{\text{on} \rightarrow \text{off}} := \{a \in \mathcal{A}^{\text{order}} | v_{a,p^*(p)} - v_{a,p} = -1 \quad \forall k \in \mathcal{K}, p \in \mathcal{P}_k^{\text{out}}\}$  to analyze which orders were given to LOS outlier patients while the reference patient didn't receive the order.

#### 4. Results

The data for this study were obtained from a pediatric hospital in Pittsburgh.  $|\mathcal{P}| = 353$  appendectomy patients were hospitalized for, on average, for 78.968 hours. Important variables extracted from the data warehouse include, among others, diagnosis codes, gender, age and 636 unique orders that were entered using Computerized Physician Order Entry. All patient-identifiable health information was removed to create a de-identified dataset for this study.

A histogram of the LOS distribution including a Gaussian kernel density curve is shown in Figure 1(a). We used Equation (1) to determine the outlier LOS threshold  $o_p$  which is 229.140 hours. The figure reveals a skewed distribution with a density maximum at the first interval. Another observation is a large proportion of patients after the outlier LOS threshold. A boxplot of the LOS is shown in Figure 1(b). One can observe that the median is very close to the first quartile and some LOS outliers can be observed after the 95 percentile.

To investigate whether a parametric model may be used to fit the data, we ran experiments with 9 distributions such as Beta, Log-normal, Weibull and Erlang. Our results revealed that the Beta distribution results in the best fit with respect to the squared error between the empirical and the best theoretical distribution. The log-normal distribution fits second best and its results of the fitting process will be analysed in more detail: Both the Chi-Square (CS) and the Kolmogorov-Smirnov (KS) test resulted in  $p < 0.01$  while the CS-test run with 7 intervals and 4 degrees of freedom resulted in a  $p < 0.005$ . The optimal parameters of the (theoretical) log-normal

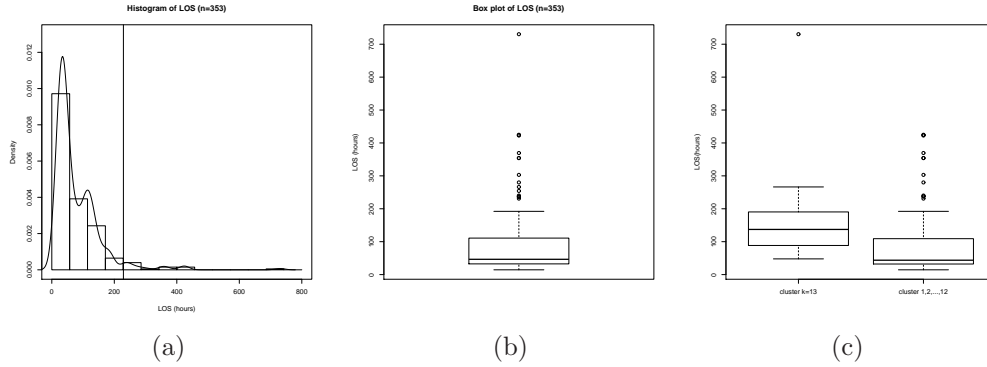


Figure 1: LOS distribution (a), LOS box plot (b) clustered patients' LOS (c)

distribution's expected value and variance come up to  $\mu = 72$  and  $\sigma^2 = 123$ , respectively with a LOS-intercept of 14 (hours) based on the empirical minimum LOS value. Using this distribution to fit our data, the squared error comes up to 0.137. The result that the log-normal distribution fits very well is not surprising and confirms assumptions from the literature, see Min and Yih (2010).

In our first stage clustering, we varied the number of  $k$  until we reached a cluster in which the outlier flag was present. The first cluster was  $k = 13$ . The clinical and demographic information are shown in Table 1(a) and a summary statistics is shown in Table 1(b). The table shows that ICD-9 code 540.1 – ‘Acute appendicitis with peritoneal abscess’, an emergency type of 4, a moderate APR DRG severity and ‘laparoscopic appendectomy’ are the attributes in which outlier patients are most likely to be present. Figure 1(c) shows a LOS boxplot of patients of which the demographic and clinical attributes belong to this cluster vs. all other patients. In a second stage, we clustered based on orders to determine the switching patterns. Again, we run the  $k$ -means algorithm and now want to discover differences in the prescription of orders. We came up with two clusters with a total number of 306 orders. Table 1(d) shows the results of the second stage clustering. The table reveals that the number of orders more than doubles from cluster  $k = 2$  to cluster  $k = 1$ . One explanation for this phenomenon is that the length of stay is longer and therefore more orders are likely to be prescribed to patients. Another observation is that in cluster  $k = 2$  the LOS more than triples as compared to cluster  $k = 1$ . Now, comparing both

clusters, we observed  $|\mathcal{A}_{13}^{\text{o, on} \rightarrow \text{off}}| = 52$  occurrences with a switch from on-off while a off-on was only observed  $|\mathcal{A}_{13}^{\text{o, off} \rightarrow \text{on}}| = 4$  times. In the latter case, we predominantly observed order switches in drug and diet prescriptions.

$\mathcal{A}$	$v_a$		
Diagnosis code	540.1	Number of Data Points	17
Emergency type	4	Min Data Value	47.7
APR DRG Severity	Moderate	Max Data Value	730
Laparoscopic appendectomy	yes	Sample Mean	178.9
		Sample Std Dev	156.5
		1 <sup>st</sup> quartile	88.5
		2 <sup>nd</sup> quartile	137.2
		3 <sup>rd</sup> quartile	190.4

(a)

(b)

Number of Data Points	336			
Min Data Value	14.5			
Max Data Value	424.9	cluster	#orders on	#orders off
Sample Mean	73.9			Mean LOS
Sample Std Dev	66.7	$k = 1$	90	216
1 <sup>st</sup> quartile	32.2	$k = 2$	42	264
2 <sup>nd</sup> quartile	43.7			
3 <sup>rd</sup> quartile	109.1			

(c)

(d)

Table 1: Outlier cluster  $k = 13$  (a), its summary statistics (b) and summary statistics of patients not belonging to it (c) and order switchings after the 2nd stage clustering (d)

As a consequence of our study, if we assume that the patient population in cluster  $k = 2$  could be moved towards the patient population in cluster  $k = 1$  through order switching, we can determine a lower LOS bound. Applied to our dataset, the total length of stay could be reduced from 78.97 to 76.11 hours which equals to a 3.8% LOS reduction. In practice and to create a decision support tool which involves clinicians, similar reference patients may be presented to a clinician when treating each particular patient. A clinician may then decide to what extent order switching is appropriate within the limits of clinical guidelines.

## 5. Summary and Conclusions

In this paper, we have developed a clustering approach of patients for the management of length of stay outliers for pediatric appendectomy. We provided a two-stage clustering method to cluster patients based on similar clinical, demographic and length of stay characteristics and applied it to a data set including more than 350 patients. We retrieved a cluster of patients in which LOS outliers are likely to occur. In a second stage, we compared order prescription for LOS outliers with the ones for patients who have similar clinical and demographic characteristics but are non-outlier patients. Future work will extend this work towards the LOS outlier management of chronic conditions such as asthma and to incorporate clinicians' feedback into our methods.

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